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Nonlinearity and Smooth Breaks in Unit Root Testing

Tolga Omay, Dilem Yıldırım

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Testing for a unit root in the presence of a nonlinear trend: The case of Australian Reel Exchange Rate Authors: Mubariz Hasanov ABSTRACT In this paper, we examine stationarity of the Australian real exchange rate (RER). For this purpose, we modify the test of Kapetanios et. al. [Testing for a unit root in the nonlinear STAR framework. Journal of Econometrics 112 (2003) 359-379] to allow for a nonlinear trend function in the data generating process. Using bootstrap techniques, we show that the null hypothesis of unit root can be rejected, providing evidence in favour of PPP proposition for the Australian RER. Keywords: Purchasing Power Parity, Nonlinearity, Unit Root JEL Codes: C12, C22, F31	10-17
A Survey about Bias Reduction in Smooth Transition Panel Data Analysis Authors: Tolga Omay ABSTRACT There is set of biases which are effected the parameter estimation of the panel data. In this study we are giving these biases as well as the remedies. There are several categorizations in the panel data estimation with respect to time and cross-section dimension. Therefore every category has its own biases depending on the time and cross-section dimension. On the other hand the dynamic structure of the panel data is another important determinant in which we can classify the biases. Hence, the static and dynamic panel data models are also discussed separately in this study. Keywords: Panel data; Bias; Large-moderate-small panel data; static panel data; dynamic panel data JEL Codes: C23; C33	18-29

Cross-section Dependency and the Effects of Nonlinearity in Panel Unit Testing**Authors: Furkan Emirmahmutoglu****ABSTRACT**

In this study, we have analyzed the Cross Section Dependency (CSD) test in panel unit root test by using the Pesaran (2004, 2008) CD tests. For this purposes we have generate linear and nonlinear data and show the effects of nonlinear modeling on cross section dependency inherited in panel analysis. The simulation study shows us that the nonlinear models remedy some part of the CSD.

Keywords: Panel Unit Root; Cross Section Dependency Bias; Cross Section Dependency Test

JEL Codes: C23;C63



Nonlinearity and Smooth Breaks in Unit Root Testing

Tolga Omay¹
Dilem Yıldırım²

Abstract

We develop unit root tests that allow under the alternative hypothesis for a smooth transition between deterministic linear trends, around which stationary asymmetric adjustment may occur by employing exponential smooth transition auto-regressive (ESTAR) models. The small sample properties of the newly developed test are briefly investigated and an application for investigating the PPP hypothesis for Argentina is provided.

Keywords: Smooth Break; Nonlinear Unit Root Test; PPP.

JEL Codes: C12; C22; O47.

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1. Introduction

In this study we have developed a test for the unit root null hypothesis by combining the methodologies developed by Kapetanios et al (2003) (henceforth KSS) and Leybourne, Newbold and Vougas (1998) (henceforth LNV). KSS (2003) employ exponential smooth transition autoregressive (ESTAR) models to propose tests of the null hypothesis of a unit root that allow under the alternative hypothesis for stationary nonlinear adjustment towards a fixed mean. Thus, we extend the KSS tests to the case of a nonlinear attractor³.

Section 2 of this paper develops the proposed test statistics and represents their critical values. Section 3 provides the small sample performance of the proposed test in comparison with the power of the ADF, LNV, Sollis, KSS and EG tests. Section 4 presents the application of our aforementioned tests to the PPP hypothesis.

2. The model and testing framework

Let y_t be a changing trend function with smooth transition on the time domain $t = 1, 2, \dots, T$.

$$y_t = \alpha + \alpha_2 S_t(\gamma, \tau) + \varepsilon_t \quad (1)$$

$$y_t = \alpha + \beta_1 t + \alpha_2 S_t(\gamma, \tau) + \varepsilon_t \quad (2)$$

$$y_t = \alpha + \beta_1 t + \alpha_2 S_t(\gamma, \tau) + \beta_2 t S_t(\gamma, \tau) + \varepsilon_t \quad (3)$$

where ε_t is a zero mean $I(0)$ process and $S_t(\gamma, \tau)$ is logistic smooth transition function, based on a sample of size T and N ,

$$S_t(\gamma, \tau) = \left[1 + \exp\{-\gamma(t - \tau T)\} \right]^{-1}, \quad \gamma > 0 \quad (4)$$

In this modeling strategy, the structural change is modeled as a smooth transition between different regimes rather than an instantaneous structural break as in Leybourne *et al.* (1996). The transition function $S_t(\gamma, \tau)$ is a continuous function bounded between 1 and 0. Thus the STR model can be interpreted as regime-switching model that allows for two regimes, associated with the extreme values of the transition function, $S_t(\gamma, \tau) = 0$ and $S_t(\gamma, \tau) = 1$, whereas the transition from one regime to the other is gradual. The parameter γ determines the smoothness of the transition, and thus, the smoothness of transition from one regime to the other. The two regimes are associated with small and large values of the transition variable $s_t = t$ relative to the threshold $c = \tau$. For the large values of γ , $S_t(\gamma, \tau)$ passes through the interval $(0, 1)$ very rapidly, and as γ approaches $+\infty$ this function changes value from 0 to 1 instantaneously at time $t = \tau T$. Therefore, if we assume that ε_t is a $I(0)$ process with zero mean, then in model 1 y_t is a stationary process around a mean which changes from initial value α_1 to final value $\alpha_1 + \alpha_2$. Leybourne *et al.* (1996) also give similar conditions for models 2 and 3. In these specifications no change and one instantaneous structural change are

³ Enders and Granger (1998) proposed unit root tests for two regime TAR model. They named the linear trend as the linear attractor. Hence, following their suggestion we called this nonlinear trend as a nonlinear attractor.

limiting cases, whereas this unit root specification which we have given in equations 1, 2 and 3 is more general, since it covers gradual structural changes as well⁴.

We establish the hypotheses for unit root testing based on equations 1, 2 and 3 as follows:

$$\begin{aligned} H_0 : & \text{Unit Root, (Linear Nonstationary)} \\ H_a : & \text{Nonlinear Stationary (Nonlinear and Stationary around smoothly} \\ & \text{changing trend and intercept)} \end{aligned} \quad (5)$$

Following Leybourne *et al.* (1996) the test statistics proposed here are calculated with a two-step procedure:

Step 1. Using a nonlinear least squares (NLS) algorithm, estimate only deterministic component of the preferred model and compute the NLS residuals

$$\begin{aligned} \text{Model 1} \quad \hat{\varepsilon}_t &= y_t - \hat{\alpha}_1 - \hat{\alpha}_2 S_t(\gamma, \tau) \\ \text{Model 2} \quad \hat{\varepsilon}_t &= y_t - \hat{\alpha}_1 - \hat{\beta}_1 t - \hat{\alpha}_2 S_t(\gamma, \tau) \\ \text{Model 3} \quad \hat{\varepsilon}_t &= y_t - \hat{\alpha}_1 + \hat{\beta}_1 t + \hat{\alpha}_2 S_t(\gamma, \tau) + \hat{\beta}_2 t S_t(\gamma, \tau) \end{aligned}$$

Step 2. Compute the KSS statistic, the t ratio associated with $\hat{\rho}_i$ in the ordinary least squares (OLS) regression

$$\Delta \hat{\varepsilon}_t = \hat{\rho} \hat{\varepsilon}_t^3 + \sum_{j=1}^k \hat{\delta}_j \Delta \hat{\varepsilon}_{t-j} + \hat{\eta}_t \quad (6)$$

For model 1, 2 and 3 we denote the t statistics for $\hat{\rho}_i$ as $\bar{t}_{br1}$, $\bar{t}_{br2}$, and $\bar{t}_{br3}$, respectively.

$$\begin{aligned} H_0 : \rho &= 0, \quad \text{for all } i, \quad (\text{Linear Nonstationary}) \\ H_0 : \rho &< 0, \quad \text{for some } i, \quad (\text{Nonlinear and Stationary around nonlinear trend and intercept}) \end{aligned}$$

Table 1. Critical Values

	Model 1			Model 2			Model 3		
	%10	%5	%1	%10	%5	%1	%10	%5	%1
25	-3.691	-4.133	-5.056	-4.296	-4.728	-5.543	-4.609	-5.048	-5.873
50	-3.521	-3.870	-4.571	-3.963	-4.327	-5.106	-4.214	-4.593	-5.380
100	-3.509	-3.821	-4.443	-3.889	-4.202	-4.777	-4.090	-4.411	-5.041
200	-3.496	-3.810	-4.424	-3.885	-4.189	-4.771	-4.062	-4.382	-4.980
500	-3.489	-3.801	-4.412	-3.879	-4.180	-4.757	-4.053	-4.370	-4.969

⁴ For further discussion and the possible extensions see Leybourne *et al.* (1996).

2.1 Finite sample performance

We have investigated the empirical power of our newly proposed test by using the following data generating process where the process is a stationary nonlinear adjustment around a smooth transition from one constant value to another. Thus, the following ST-ESTAR(1) was employed as a DGP:

$$y_t = \alpha_1 + \alpha_2 S_t(\gamma, \tau) + \varepsilon_t, \quad \mu_0 = 0$$

$$\Delta \varepsilon_t = \alpha + \gamma \varepsilon_{t-1} \left(1 - \exp[-\theta \varepsilon_{t-1}^2]\right) + \eta_t, \quad \eta_t \sim NID(0,1)$$

where S_t is defined as before, and all combinations of the following parameter values were used; two extreme values for the gamma parameter $\gamma = -0.1, -1.0$, two extreme values for the transition speed parameter $\theta = 0.01, 1.0$ and small and large values for the structural break parameter $\alpha_2 = 2.0, 10.0$. The results from these power experiments for a sample size of $T = 100$ are given in Table 2.1.1.

Table 2.1.1 The power comparison of alternative tests

α_2	λ	c	θ	γ	$S_{\alpha, NL}$	s_α	t_{s_α}	F_α	t_{NL}	$T_{\max, t}$	Φ_t	τ_μ
2.0	0.5	0.2	0.01	-0.1	0.062	0.046	0.046	0.034	0.062	0.064	0.048	0.044
2.0	0.5	0.2	1.0	-0.1	0.086	0.076	0.070	0.068	0.190	0.144	0.162	0.096
2.0	0.5	0.2	0.01	-1.0	0.164	0.130	0.148	0.116	0.234	0.176	0.208	0.188
2.0	0.5	0.2	1.0	-1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.0	0.5	0.5	0.01	-0.1	0.070	0.064	0.064	0.052	0.086	0.080	0.074	0.040
2.0	0.5	0.5	1.0	-0.1	0.100	0.092	0.084	0.086	0.148	0.110	0.134	0.100
2.0	0.5	0.5	0.01	-1.0	0.152	0.132	0.110	0.108	0.208	0.152	0.180	0.152
2.0	0.5	0.5	1.0	-1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.0	5.0	0.2	0.01	-0.1	0.050	0.038	0.042	0.026	0.078	0.068	0.068	0.040
2.0	5.0	0.2	1.0	-0.1	0.116	0.110	0.092	0.098	0.206	0.142	0.174	0.100
2.0	5.0	0.2	0.01	-1.0	0.160	0.130	0.126	0.124	0.204	0.174	0.190	0.142
2.0	5.0	0.2	1.0	-1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.0	5.0	0.5	0.01	-0.1	0.052	0.042	0.040	0.032	0.092	0.060	0.074	0.060
2.0	5.0	0.5	1.0	-0.1	0.096	0.118	0.088	0.098	0.200	0.120	0.164	0.082
2.0	5.0	0.5	0.01	-1.0	0.190	0.162	0.130	0.150	0.236	0.164	0.212	0.174
2.0	5.0	0.5	1.0	-1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
10.0	0.5	0.2	0.01	-0.1	0.026	0.012	0.016	0.008	0.006	0.020	0.002	0.018
10.0	0.5	0.2	1.0	-0.1	0.076	0.068	0.052	0.054	0.006	0.020	0.000	0.022
10.0	0.5	0.2	0.01	-1.0	0.120	0.080	0.060	0.074	0.016	0.026	0.012	0.024
10.0	0.5	0.2	1.0	-1.0	1.000	1.000	1.000	1.000	0.008	0.038	0.002	0.231
10.0	0.5	0.5	0.01	-0.1	0.042	0.042	0.040	0.034	0.014	0.010	0.004	0.014
10.0	0.5	0.5	1.0	-0.1	0.114	0.088	0.068	0.072	0.014	0.012	0.004	0.012
10.0	0.5	0.5	0.01	-1.0	0.242	0.161	0.191	0.111	0.030	0.020	0.010	0.030
10.0	0.5	0.5	1.0	-1.0	1.000	1.000	1.000	1.000	0.072	0.058	0.042	0.318
10.0	5.0	0.2	0.01	-0.1	0.114	0.028	0.034	0.016	0.010	0.032	0.002	0.066
10.0	5.0	0.2	1.0	-0.1	0.144	0.074	0.068	0.068	0.004	0.028	0.004	0.076
10.0	5.0	0.2	0.01	-1.0	0.188	0.070	0.086	0.064	0.004	0.024	0.000	0.064
10.0	5.0	0.2	1.0	-1.0	1.000	1.000	1.000	1.000	0.004	0.036	0.002	0.434
10.0	5.0	0.5	0.01	-0.1	0.160	0.060	0.096	0.062	0.026	0.032	0.018	0.064
10.0	5.0	0.5	1.0	-0.1	0.274	0.148	0.180	0.144	0.016	0.024	0.008	0.052

10.0	5.0	0.5	0.01	-1.0	0.398	0.190	0.198	0.182	0.020	0.032	0.014	0.068
10.0	5.0	0.5	1.0	-1.0	1.000	1.000	1.000	1.000	0.058	0.070	0.038	0.282

Note 1: Leybourne *et al.* (1996) stated that Model 1's natural competitor is the ADF test that involves both an intercept and trend term. Therefore, we compare the Model 1 proposed in this paper with the KSS, ADF and EG tests.

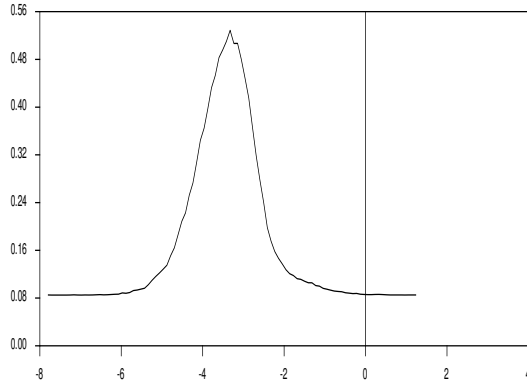
Note 2: $S_{\alpha,NL}$, S_{α} , $t_{s_{\alpha}}$ and F_{α} denote the proposed test, LNV, Solis max-t and F tests, respectively. t_{NL} , T_{max_t} , Φ_t and τ_{μ} denote the KSS, EG max-t, EG F and DF tests, respectively. The second group of tests does not cover the structural break in their testing procedure.

For a small break ($\alpha_2 = 2.0$), the power of the KSS test exceeds that of the newly proposed test. However, as expected, in the large break case ($\alpha_2 = 10.0$) the newly proposed test over performs all of the tests in all parameter regions.

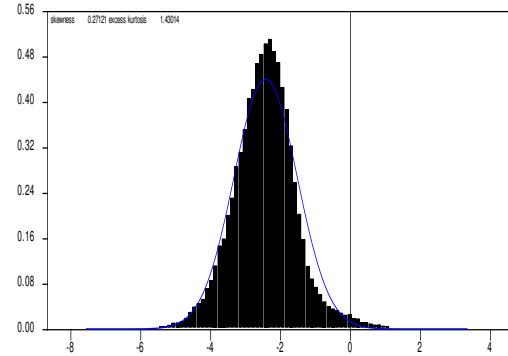
2.2 The Asymptotic Properties of the Proposed Test

Leybourne *et al.* (1996) state that as NLLS estimation of the parameters gamma (γ) and tou (τ) does not admit closed form solutions, it would be extremely difficult to subsequently establish any analytical relationship between the residual terms that are obtained from the STR estimation of the deterministic component and the dependent variable. Therefore, this makes the determination of the null asymptotic distribution of the test statistics by analytical means more or less intractable. Moreover, in our testing procedure we are introducing another form of nonlinearity around the deterministic component which makes it harder to obtain the asymptotic distribution. Thus, we use the simulation methodology to examine the nonlinear asymptotic relationship. In order to see the difference between normal distribution and the distribution that we obtained from our test statistics, we have obtained density function of the test statistics. The below figures are the simulation results:

The simulated density of test statistics for T= 50



The comparison with normal distribution



Observations	50000
Sample Mean	-2.415
Variance	0.822
Standard Error	0.906
Skewness	0.271 (0.000)
Kurtosis (excess)	1.430 (0.000)
Jarque-Bera	4874.005 (0.000)

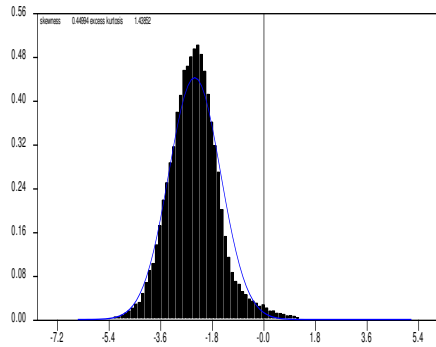
*The solid line is normal distribution and the shaded area is simulated density function of the proposed test

* Number of Simulation 50000

* Time series dimension T = 50

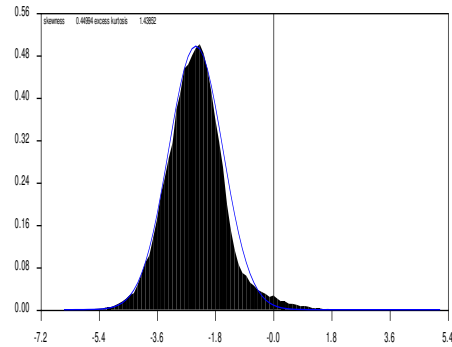
Figure 1. The simulated density of test statistics and comparison with normal distribution for T: 50

The simulated density of test statistics



Sample Mean	-2.400
Variance	0.815
Standard Error	0.903
Skewness	0.449 (0.000)
Kurtosis (excess)	1.438 (0.000)
Jarque-Bera	5998.221 (0.000)

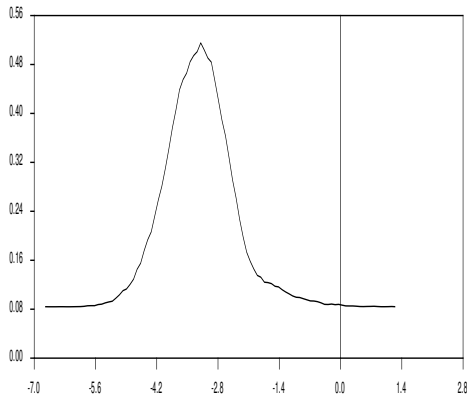
The comparison with normal distribution



*The solid line is normal distribution and the shaded area is simulated density function of the proposed test
 * Number of Simulation 50000
 * Time series dimension T = 100

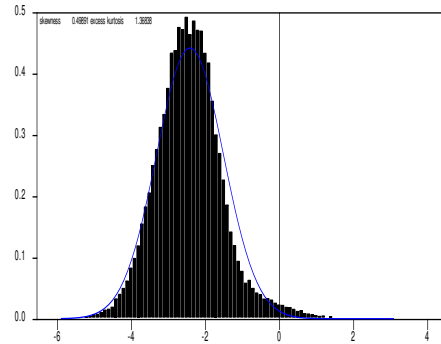
Figure 2. The simulated density of test statistics and comparison with normal distribution for T: 100

The simulated density of test statistics for



Sample Mean	-2.413
Variance	0.821
Standard Error	0.906
Skewness	0.506 (0.000)
Kurtosis (excess)	1.408 (0.000)
Jarque-Bera	6271.965 (0.000)

The comparison with normal distribution



*The solid line is normal distribution and the shaded area is simulated density function of the proposed test
 * Number of Simulation 50000
 * Time series dimension T = 500

Figure 3. The simulated density of test statistics and comparison with normal distribution for T: 500

3. Empirical Example

In this section we empirically apply all the unit root tests utilized in the power analysis to examine the validity of the purchasing power parity (PPP) hypothesis for Argentina over the period 2003:6-2011:10. Monthly data on the bilateral exchange rate of the national currency against the U.S. dollar and on consumer price indices (CPI) was taken from International Monetary Fund's *International Financial Statistics* (IFS) database. The base year for the CPI is 1997. All variables were put into natural logarithms before the analysis.

Table 11. The PPP hypothesis under alternative unit root tests.

	$S_{\alpha,NL}$	s_{α}	F_{α}	t_{NL}	Φ_t	τ_{μ}
Argentina	-7.315	-2.156	2.325	-2.398	1.813	-1.770

Note: LNV %10 %5 and %1 significance level -3.909, -4.232,-4.882, Sollis %10 %5 and %1 significance level 7.844, 9.191,12.244. KKS %10 %5 and %1 significance level -2.66, -2.93,-3.48. EG %10 %5 and %1 significance level 3.79, 4.64 ,6.57. ADF 10 %5 and %1 significance level -2.58, -2.89, -3.51

The results of the ADF, PP, KSS, EG and Sollis unit root tests recommend that the null hypothesis of a unit root is rejected at the conventional significance levels. These results contradict the PPP hypothesis. On the other hand, our newly proposed test that allows for nonlinear adjustment towards LNV type trend function rejects the null hypothesis of a unit root at 1% significance level, which provides an evidence for the PPP hypothesis. This finding recommends that a model that allows for gradual structural breaks and nonlinear adjustment might be more suitable for the Argentinian RER series.

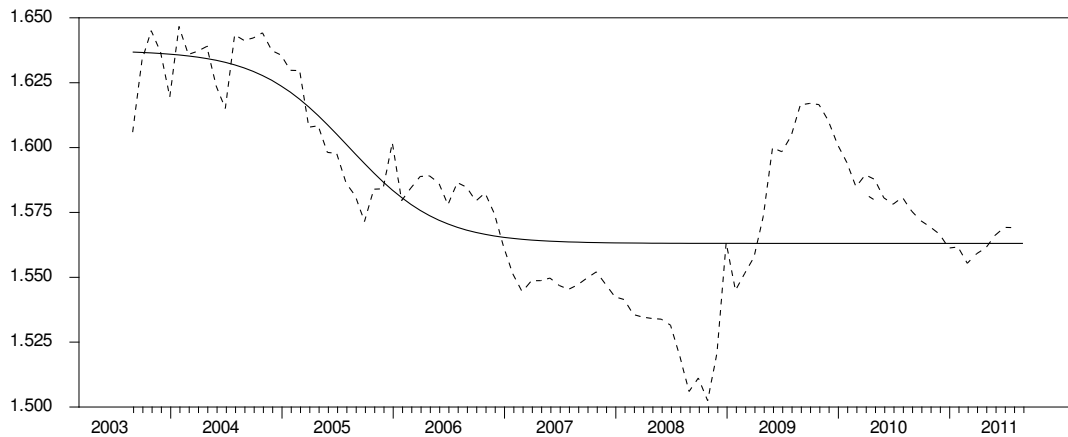


Figure 1. Estimated STR type trend functions (Model A) for Argentina

4. Conclusion

In this study, we have proposed a nonlinear unit root which also considers structural break. By using this newly proposed test we show the validity of the PPP hypothesis for the Argentinian real exchange rate series.

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**Testing for a unit root in the presence of a nonlinear trend:
The case of Australian Reel Exchange Rate**

Mübariz Hasanov¹

Abstract

In this paper, we examine stationarity of the Australian real exchange rate (RER). For this purpose, we modify the test of Kapetanios et. al. [Testing for a unit root in the nonlinear STAR framework. Journal of Econometrics 112 (2003) 359-379] to allow for a nonlinear trend function in the data generating process. Using bootstrap techniques, we show that the null hypothesis of unit root can be rejected, providing evidence in favour of PPP proposition for the Australian RER.

Keywords: Purchasing Power Parity, Nonlinearity, Unit Root

JEL Codes: C12, C22, F31

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1. Introduction

Purchasing power parity (PPP) is, perhaps, one of the most important propositions in international economics literature. According to the PPP proposition, nominal exchange rates move together with differences in relative prices in two countries. While there may be small deviations in relative prices due to transportation costs and trade barriers, it is generally agreed that large deviations in relative prices will be traded away by arbitrageurs, and real exchange rates will tend toward PPP in the long run (Rogoff 1996).

PPP suggests that real exchange rate follows a stationary process. Therefore, tests of PPP have often been based on examination of stochastic properties of real exchange rate series. Earlier researchers that used conventional cointegration tests failed to find stationarity, contradicting to PPP. Such a failure was attributed to low power of cointegration tests, and researchers have been searching for alternative frameworks within which to test for unit roots. For example, Frankel and Rose (1996) applied panel unit-root tests to increase the number of observations. However, Taylor et al. (2001) argues that conclusions drawn from panel unit root tests may be misleading owing to incorrect interpretation of the null hypothesis of joint non-mean reversion of all series under investigation. Another strand of the literature points to possible nonlinearities in real exchange rate dynamics and argues that the PPP puzzle can be resolved once nonlinear adjustment toward equilibrium is properly modelled (e.g. Michael et al., 1997; Taylor et al. 2001). On the other hand, Lundbergh et al. (2003) argued that time series might be described more appropriately by simultaneous structural break and nonlinearities. In fact, Telatar and Hasanov (2009), using the test procedure of Sollis (2004), which allows for both a structural break and asymmetric adjustment in the data generating process, provide empirical evidence in favour of PPP in selected transition countries.

Taking account of the fact that both structural breaks and nonlinearities may characterize real exchange rate dynamics, in this paper we propose an alternative method to test for a unit root. Mainly, we consider a possibility that real exchange rate series may follow a nonlinear stationary process around a deterministic (nonlinear) trend function. For this purpose we generalize the test procedure of Kapetanios et al. (2003), henceforth KSS, to allow for nonlinear trend function. KSS develop unit root test procedure against globally stationary exponential smooth transition (ESTAR) type nonlinearity, which is thought to be relevant for real exchange rate dynamics (see, for example, Michael et al. 1997; Taylor et al. 2001; Kapetanios et al. 2003). Unlike KSS, who consider only a linear trend function, we allow for a polynomial trend function in the data generating process. Using bootstrap critical values, we show that the Australian real exchange rate (RER)² is stationary around nonlinear trend function.

The rest of the paper is organized as follows. In section 2 we briefly discuss the econometric methodology. In section 3 we present the empirical results and then section 4 concludes.

² Stationarity of the Australian RER has previously been examined in the empirical literature intensively. See, for example, Corbae and Ouliaris (1990), Henry and Olekalns (2002), Darné and Hoarau (2008), Cuestas and Regis (2008) and Cuestas and Gil-Alana (2009), who provide mixed evidence on the validity of the PPP for Australia. For a critique of previous works on stationarity of the Australian RER see Cuestas and Gil-Alana (2009).

2. Econometric Methodology

Following Chortareas et al. (2002), we consider following data generating process:

$$y_t = \mu + \phi(t) + x_t, \quad t = 1, \dots, T \quad (1)$$

$$\Delta x_t = \gamma x_{t-1} \{1 - \exp(-\theta x_{t-1}^2)\} + \varepsilon_t, \quad (2)$$

where $-2 < \gamma < 0$, and $\varepsilon_t \sim iid(0, \sigma^2)$. As shown by KSS, one may test for unit root by testing the null hypothesis $\theta = 0$ against the alternative hypothesis $\theta > 0$. Under the null hypothesis y_t follows a linear unit root process around deterministic trend function $\phi(t)$, whereas it is nonlinear stationary ESTAR process under the alternative³.

The above representation of the data generating process has a nice property in that, it allows for deterministic trend function under both the null hypothesis and the alternative hypothesis, without introducing any parameters that are irrelevant under either (Schmidt and Phillips, 1992). Chortareas et al. (2002) assume that the trend function $\phi(t)$ in Eq. (1) is linear, and propose alternative de-trending methodology for the KSS test following Schmidt and Phillips (1992).

Different specifications for the trend function $\phi(t)$ have been considered in tests of unit root. For example, Leybourne et al. (1998) and Sollis (2004) use logistic trend function that allows for a smooth break in the trend. Bierens (1997) models nonlinear trend using Chebyshev polynomials. Becker et al. (2006) use trigonometric functions (via means of Fourier transformations) to model possible gradual breaks in the data generating process. Instead of imposing specific form of nonlinearity, following Ouliaris et al. (1988) and Schmidt and Phillips (1992), in this paper we use time polynomials to model possible nonlinearities in the trend function. Use of time polynomials to model nonlinear trends has advantages over other specifications. First, unlike logistic functions, time polynomials do not impose specific form of nonlinearity in the trend function. In addition, as shown by Lin and Terasvirta (1994), first and higher order logistic transition functions in time trend can well be approximated by time polynomials. Second, use of trigonometric functions or Chebyshev polynomials might be problematic. As Cuestas and Gil-Alana (2009) argue, Chebyshev polynomial approximation for the nonlinear time trend is problematic because there is no unique way of selection of the order of the polynomials. A similar problem arises in selecting frequency components for trigonometric functions (Becker et al. 2006). However, in the case of time polynomials, one may use conventional statistical significance tests to choose the order of time polynomials, without any burden on estimating various nonlinear models and choosing the appropriate one. Therefore, use of time polynomials allows one to model more general classes of nonlinearities and possible multiple structural breaks in the trend function, and to choose appropriate order of polynomials.

Although Chortareas et al. (2002) use different de-trending methodology following Schmidt and Phillips (1992), in this paper we follow KSS to de-trend the series under investigation. Assuming that the nonlinear trend function can be approximated by a time polynomial, Eq. (1) can be re-written as follows:

$$y_t = \mu + \sum_{i=1}^p \beta_i t^i + x_t \quad (3)$$

³ See Kapetanios et. al. (2003) for further discussions on geometric ergodicity and asymptotic properties of the ESTAR models.

The order of time polynomial can be selected by testing significance of additional powers of time trend. Define de-trended series $x_t = y_t - \hat{\mu} - \sum_{i=1}^p \hat{\beta}_i t^i$ where $\hat{\mu}$ and $\hat{\beta}_i$ are OLS estimates of μ and β_i , $i=1, \dots, p$. Now, one may test stationarity of de-trended series x_t following KSS. As KSS argue, testing the null hypothesis $H_0: \theta=0$ directly is not feasible because the parameter γ is not identified under the null. Therefore, one may follow Luukkonen et al. (1988) and replace the exponential transition function with its first order Taylor approximation, which gives the following auxiliary regression for the de-trended series x_t :

$$\Delta x_t = \delta x_t^3 + e_t \quad (4)$$

where e_t comprises original shocks ε_t as well as the error term resulting from Taylor approximation. The auxiliary regression model (4) suggests that one may obtain t -statistic for $\delta=0$ against $\delta < 0$ as follows:

$$t_{NTNL} = \hat{\delta} / s.e.(\hat{\delta}), \quad (5)$$

where $\hat{\delta}$ is the OLS estimate and $s.e.(\hat{\delta})$ is the standard error of $\hat{\delta}$.

To deal with possible serial correlation in error term ε_t in (2), one may augment the auxiliary regression (4) with lagged values of Δx_t :

$$\Delta x_t = \delta x_{t-1}^3 + \sum_{j=1}^p \alpha_j \Delta x_{t-j} + e_t \quad (6)$$

Unlike the case of testing linearity against STAR-type nonlinearity, the t_{NTNL} test does not have an asymptotic standard normal distribution (e.g., Chortareas et al., 2002; Kapetanios et al., 2003). Therefore, we obtain empirical distribution of the test statistic the t_{NTNL} using bootstrap techniques.

3. Empirical Results

In this paper we use quarterly observations of the Australian real Trade Weighted Index (TWI) computed by the Reserve Bank of Australia. The sample covers the period from 1970:Q2 to 2009:Q4. In the first step, we estimate equation (3). The order of time polynomial was selected by testing significance of additional powers of time trend using maximum order of 12. Figure 1 plots real exchange rate series along with estimated nonlinear trend function and deviations from the trend⁴. The estimated nonlinear trend function appears to be a good approximation for the deterministic component of the series and suggests that structural changes were rather gradual.

⁴ Selected order of time polynomials is 10. The estimates of coefficients of time polynomials are available upon request.

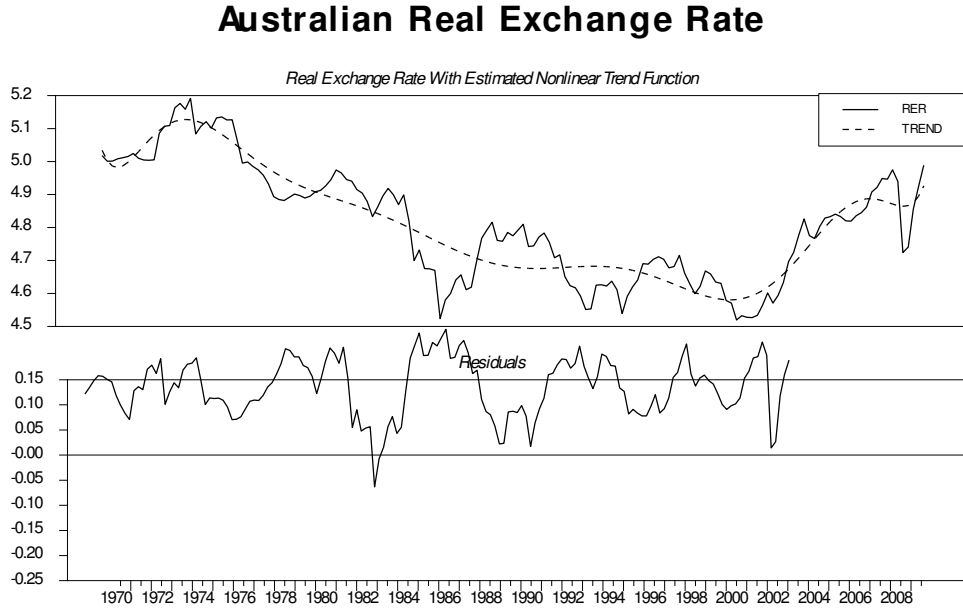


Figure 1. Australian Real Exchange Rate with Estimated Nonlinear Trend Function

In the next step, using the residuals from equation (3) we estimate equation (6) to compute the test statistics t_{NTNL} . The lag order in equation (6) was selected so as to remove significant autocorrelation in the residuals. For comparison purposes, in addition to the proposed test we have also performed conventional ADF (Dickey and Fuller, 1979), Phillips-Perron (1988) and KPSS (Kwiatkowski et al., 1992) stationarity tests as well as KSS nonlinear unit root test (2003) and Sollis's (2004) ST-TAR unit root test that allows for simultaneous smooth structural change and asymmetric adjustment in the data generating process. The results of these tests are provided in Table 1.

Table 1 Unit Root Test Results

	ADF	PP	KPSS	KSS	ST-TAR	t_{NTNL}
Test statistics	-1.312	-1.083	0.276*	-2.868	-2.987	-3.982**
Critical Values						
1%	-4.020	-4.018	0.216	-3.93	-4.393	-4.458
5%	-3.440	-3.439	0.146	-3.40	-3.937	-3.775
10%	-3.144	-3.143	0.119	-3.13	-3.704	-3.419

Notes: All test regressions include time trend. * and ** denote rejection of the null hypothesis at 1% and 5% significance levels, respectively. The KPSS test differs from other tests in that it assumes that the series under investigation are stationary under the null hypothesis. The bootstrap critical values for the t_{NTNL} statistics were calculated using 100,000 replications.

The results of the ADF, PP, KSS and ST-TAR unit root tests suggest that the null hypothesis of unit root is rejected at conventional significance levels. Similarly, the KPSS test rejects the null hypothesis of stationarity at 1% significance level, contradicting to the PPP proposition. However, our new proposed test that allows for nonlinear adjustment towards nonlinear trend function rejects the null hypothesis of unit root at 5% significance level,

providing some evidence in favour of PPP for the Australian RER. This finding suggests that a model that allows for more gradual structural breaks and nonlinear adjustment (rather than single structural break and nonlinear models) might be more appropriate for the Australian RER series.

4. Conclusion

In this paper, we tested the validity of PPP for the Australian real exchange rate series. Conventional unit root tests as well as newly developed test procedures that allow for nonlinearity and single gradual structural break fails to reject the null hypothesis of unit root, contradicting to the PPP proposition. However, after allowing for higher order nonlinearity in the deterministic trend function (which may capture several gradual structural breaks) as well as nonlinear adjustment in the series, we were able to reject the null hypothesis of unit root in the Australian RER series, providing some evidence in favour of the PPP proposition.

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A Survey about Smooth Transition Panel Data Analysis

Tolga Omay¹

Abstract

This study introduces a literature survey about the panel smooth transition regression models. This type of modeling emerged from two different strand of literature where the first is nonlinear time series the other is the panel data analysis. Both of these fields dealt with different type of biases in estimation process. Therefore, combining these two fields constitutes different problems in estimation. Hence, instead of giving the studies in chronological order, explanation of papers with respect to problems they have solved is preferred. Within this order, first the categories of different models are analyzed. For instance, there are several categorizations in the panel data estimation with respect to time and cross-section dimension. Therefore every category has its own biases depending on these. On the other hand the dynamic structure of the panel data is another important determinant in which classifies the biases. Hence, the static and dynamic panel smooth transition models are also discussed separately in this study. Finally, smooth transition models has its' own categories, hence these categories under the panel categorization are given as well.

Keywords: Panel smooth transition data; Bias; Large-moderate-small panel data; static panel data; dynamic panel data, Logistic, exponential, time varying

Jel Classification:

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1. Introduction

In the last two decades the availability of large data set leads researchers to analyze economical problems in panel data settings. The initiation of panel data analysis first started with small time dimension where available data was very restricted. The technological improvements and the regularities in collecting data leads to larger data sets with respect to country, firms and other entities. On the other hand the sustainability of this process increases the availability of data through large time dimensions. Hence, researchers now have ability to apply old empirical studies such as cross-sectional and time series studies in panel data settings. However, the panel data studies have typical problems. In literature, the cross-section and time series analysis biases are usually solved for linear case and can be found in text books. The linear panel data analysis problems are also discussed in text books, but the point which is mentioned above regarding the data availability is increasing. Moreover, the new category of panel data analysis emerges when the new type of biases are introduced into the panel data analysis such as cross section dependency. Cross section dependency is mainly a problem of large time dimension in the panel data analysis. Therefore it is easy to understand that this is a recent problem of panel data analysis. The first cross section dependency test is proposed by Pesaran (2004).

This study discusses different categories of panel data analysis. The first categorization depends on N and T dimension; as Large N and Small T (Type 1), Small N and Large T (Type 2) and Large N and Large T (Type 3). The second categorization is the time structure which are static and dynamic panel models. First static case and the three types of panel data models are taken into consideration. Then the dynamic structure is introduced and biases in this setting is endeavored. In panel data models, the biases and the remedies are changed due to model type and time structure.

In rest of the paper, section 2 gives the biases and the remedies of static panel data model and these are discussed in this section as well. In section 3, the biases and the remedies of dynamic panel data model are considered. Finally section 4 concludes.

2. Static panel data models biases and remedies

Panel estimators utilize the concept of combining data from different groups. One can improve efficiency in parameter estimation if there are similarities in the process generating the data in the different groups (Smith and Fuertes, 2010). Smith and Fuertes, (2010) also state that panel data estimation establishes flexibility by how parameter heterogeneity is identified over time and units. The parameters could be considered as fixed or random, homogenous or heterogeneous, and parameter variation over time periods or both could be allowed. Nevertheless, the relative magnitudes of N and T are one of the major categorization of panel data estimation. The categories can be classified as Large N and Small T (Type 1), Small N and Large T (Type 2) and Large N and Large T (Type 3). All these categories have its own estimation strategies and working assumptions.

Type 1 category is most traditional approach in panel data estimation depending on data availability. In the static case, where the regressors are strictly exogenous and the coefficients differ randomly and distributed independently of the regressors across groups, any type of averaging such as pooling, aggregating, averaging group estimates, and cross-section regression provide consistent (and unbiased) estimates of the coefficient means

(Pesaran and Smith, 1995)². Pesaran and Smith (1995) have proven that parameter heterogeneity is critical for estimating dynamic panel estimation. Hereafter, this problem is called as “Heterogeneity Bias”. Although, increasing the time periods lead to heterogeneity bias, this is not a problem in static case as mentioned above³.

Furthermore in the presence of cross-sectional correlated error terms, traditional OLS-based estimation is inefficient and invalidates much inferential theory of panel data models which is mentioned before. The traditional remedy, SURE-GLS (Seemingly Unrelated Regression Equations and Generalized Least Squares) is feasible when the cross-section dimension N is smaller than the time series dimension T . The standard approach is treating equations from different cross-section units as a system of seemingly unrelated regression equations and then estimate the system by GLS technique. If both of these dimensions are same, then the disturbance covariance matrix will be rank deficient. However, when the non-zero covariance is between the errors of different cross-section units due to common omitted variables, it is not apparent that SURE-GLS is always the correct response (Coakley *et al.*, 2002). For this reason, researcher should search for a more efficient method which is not subject to these kinds of problems. Therefore, it is apparent to use Pesaran (2006) model which suggests a method that makes use of cross-sectional averages to provide valid inference for stationary panel regressions with multifactor error structure. CCE procedure is applicable to panels with single or multiple unobserved factors so long as the numbers of unobserved factors is fixed. Groote and Everaert (2010) suggest that the PCCE estimator is quite useful for estimating cross-sectional dependent panel data models provided T is not too small. Furthermore, Pesaran (2006) has shown by Monte Carlo experiment that the simulation results favor the CCEP estimator for small to moderate sample sizes and slightly favor CCEMG when N and T are relatively large. Omay and Kan (2010) suggest a new approach by noting that the linear and non-linear combinations of the observed factors can be well approximated by cross-section averages of the dependent, independent and state dependent variable and proposed nonlinear mean group common correlated effect estimator (CCMG) for static panel. The estimation procedure has the advantage that it can be computed by least squares to auxiliary regression where the observed regressors augmented with cross-sectional averages of dependent variable, individual specific regressors and state dependent variable. This leads to a new set of estimators, which is mentioned in Pesaran (2006), referred to as the Common Correlated Effects (CCE) estimators, that can be computed by running smooth transition panel regressions augmented with cross-section averages of the dependent, independent and state dependent variables (Omay and Kan, 2010).

In the nonlinear panel literature, we have seen different types of panel smooth transition models depending on sample size and dynamic structure. Gorgens *et al.* (2009) propose nonlinear dynamic panel model in the STR context by using the Type 1 category approach. Their model is Arrelano-Bond type of panel extension of STR modelling. Hence their model has some shortcomings which are inherited in linear model like not taking in consideration of cross section dependency and the one which is mentioned above. Furthermore they find estimation of the parameters in the transition function can be problematic but there may be significant benefits in terms of forecast performance. On the

² Zellner (1969) has proven this issue by using asymptotic theory.

³ In Pesaran and Smith (1995) heterogeneity comes from the random coefficient models. Nonlinear estimation procedure Gonzales *et al.* (2005) may capture these random coefficients in two different regimes. However, we know that the nonlinear estimation which is explained above can be interpreted as heterogeneous panel estimation. Gonzales *et al.* (2005) interpreted nonlinear model as heterogeneous and linear model as homogenous. In this respect this type of estimation limits or totally eliminate the heterogeneity in estimation procedure in dynamic panel as well.

other hand, Fok *et al.* (2003) proposed a dynamic PSTR model that can be positioned between a fully pooled model, which imposes such common features, and a fully heterogeneous model, which might render estimation problems for some of the panel members. Moreover, Fok *et al.* (2003) claims that fully pooling is not effective in understanding possible common non-linear features across the series, and that a fully heterogeneous model introduces estimation problems which they have faced in their empirical research that for 4 out of the 18 series are not convergent by means of STR estimation. Furthermore, they document a partially heterogeneous panel STAR model outperforms a fully pooled model, leading to subtle differences across sectors in leads and lags for business cycle recessions and expansions. However Omay and Kan (2010) have proposed PSTAR model estimation which remedies cross section dependency. In this modeling strategy first they have estimated the nonlinear panel by using Gonzales *et al.* (2005) type PSTR model. By using this methodology they found that cross section dependency problem still could not be handled. Hence, they used the Pesaran (2006) Common Correlated Effects (CCE) estimation in nonlinear context in order to handle this. The CCE estimator is a fully heterogeneous panel time series model which augments means of dependent and independent variables in order to remove factors from error term. This factor removed remedies from the cross section dependency problem due to panel estimation. Consequently, Omay and Kan (2010) employ a method which is mentioned by Fok *et al.* (2003). However, they have not encountered a convergence problem in time series estimations which hinders fully heterogeneous nonlinear panel estimation. Fully heterogeneous panel estimation is more suitable for Type 2 panel models and smaller the N, larger the time series dimension feasibility increases. Increasing time series dimension precludes convergence problems in time series models. Fok *et al.* (2003) has used a sample period 1972Q1-2002Q4 which leads to 120 data points. The dimension of T for their empirical research is satisfactorily fulfills large T condition. However, they used 18 sectors which can cause a problem with respect to small N criteria. In this study, 8 countries covering the period 1993 to 2008 which satisfies the Small N but not large T conditions is used. Therefore, estimating fully heterogeneous panel time series model is not appropriate depending on the arguments mentioned above and sample structure. Furthermore, Omay and Kan (2010) claim that the CCE estimation removes the endogeneity bias depending on the model structure used in their estimation. On the other hand, Fouquau *et al.* (2008) estimated static PSTAR model using the Gonzales *et al.* (2005) methodology. They have shown from the parameter estimation the nonlinear model limits the potential endogeneity bias. From these arguments nonlinear estimation and CCE estimator both reduce the potential endogeneity bias in estimation procedure. Hence, this estimation process does not deal with the endogeneity bias in Nonlinear Static Pool Common Correlated Effect (NPCCE) estimation⁴.

⁴ For PCCE estimation Sarafidis and Yamagata (2010) investigate the properties of instrumental variable estimation and compare their estimators with GMM. However, they are dealing with the linear model where this estimator is more efficient and constitute consistent estimators.

Table 2. Biases and Remedies in Static Panel Data

<u>Biases in Static Panel</u>		
Heterogeneity	CSD*	Endogeneity
<u>Remedies</u>		
MG, PMG, RCM Pool FE, RE**	CCE, SURE-GLS***	IV, GMM
For N small T moderate or large		
Not Prevail: X	Prevail: $\sqrt{\quad}$	Prevail: $\sqrt{\quad}$
Nonlinear Panel estimation		
X	$\sqrt{\quad}$	$\sqrt{\quad}$ ****
Nonlinear CCE Panel estimation		
X	X	X *****

*CSD denotes Cross Section Dependency

** We can include Swamy model as well. MG denotes mean group, PMG pooled mean group, RCM denotes random coefficient model.

*** There are other methods such as time effects/ demeaning, orthogonization, PANIC Panel Analysis of Non-stationarity in the Idiosyncratic and Common components and residual principal component methods

**** $\sqrt{\quad}$ means nonlinear model partially correct endogeneity bias (Fouquau et al. 2008).

***** CCE estimation further corrected this bias Omay and Kan (2010).

2. Dynamic panel data models biases and remedies

In the panel data literature, there are several methods for handling estimation process. Pesaran and Smith (1995) compare these estimation methods extensively for dynamic panels by using asymptotic theory and conclude that the Mean Group (MG) estimation is the most efficient compared to other methods which consist different versions of implicit averaging where the MG estimation did explicitly⁵. The widely used four procedures are pooling, aggregating, averaging group estimates and cross-section regression. (MG) and Pooled Mean Group (PMG) estimations are classified as averaging group estimates, Dynamic Panel GMM estimation of Arrelano and Bond (1991) and the other estimation procedures which are using this kind of estimation procedure are classified as pooling. This study does not consider time aggregation and cross-section regression for two purposes. The first one is Pesaran and Smith (1995) found out that these estimation procedures are asymptotically inconsistent estimates. These estimators are more efficient in different cases. The relative magnitudes of N and T are one of the major categorization of panel data estimation. The categories can be classified as Large N and Small T (Type 1), Small N and Large T (Type 2) and Large N and Large T (Type 3). All these categories have its own estimation strategies and processing assumptions.

Type 1 category is most traditional approach in panel data estimation depending on data availability. In this type the classical estimator for estimation of pooled models depending on Nickell (1981) bias such as fixed effects instrumental variables or Generalized Method of Moments (GMM) estimators proposed, for example, by Anderson and Hsiao (1981, 1982), Arellano (1989), Arrelano and Bover (1995), Keane and Runkle (1992) and

⁵The aggregation method which is proposed by Zellner (1969) for random coefficient (with lagged dependent variable) model is suitable for static models. Pasaran and Smith (1995) proved that this aggregation is not prevail in dynamic panel models.

Ahn and Schmidt (1995). Pesaran and Smith (1995) show that these estimators can produce inconsistent and potentially very misleading estimates in fact the slope coefficients are identical. Therefore, Pesaran and Smith (1995) have proven that the parameter heterogeneity is critical for estimating dynamic panel estimation. Hereafter, this problem is called as “Heterogeneity Bias”. Alvarez and Arrelona (2003) have shown that the OLS estimates are consistent if time dimension grow sufficiently fast relative to N such that $N/T \rightarrow \kappa$ where $0 \leq \kappa < \infty$. However, increasing the time periods lead to Heterogeneity Bias, hence this bias can prevail in most of the nonlinear panels. In the nonlinear estimation procedure and cross section dependency part this phenomenon is re-analyzed. However, it is known that the nonlinear estimation which is explained above can be interpreted as heterogeneous panel estimation. Gonzales *et al.* (2005) interpreted nonlinear model as heterogeneous and linear model as homogenous. In this respect this type of estimation limits or totally eliminate the heterogeneity in estimation procedure⁶.

Furthermore in the presence of cross-sectionally correlated error terms, traditional OLS-based estimation are inefficient and invalidates much inferential theory of panel data models which is mentioned before. In Omay *et al.* (2012), a system SURE-GLS is used in order to eliminate cross section dependency and endogeneity bias together. Depending on Coakley *et al.* (2002) system SURE-GLS approach is not seem to be that much feasible with respect to cross section dependency bias. In the static case it is mentioned that the other estimation strategy in order to eliminate cross section dependency bias is CCE estimator which is proposed by Pesaran (2006). Pesaran (2006) model which suggested a method that makes use of cross-sectional averages to provide valid inference for stationary panel regressions with multifactor error structure. Omay *et al.* (2012) extend Omay and Kan (2010) approach to dynamic nonlinear panel models. Omay and Kan (2010) proposed nonlinear mean group common correlated effect estimator (MGCCE) for static panel where they employ this estimator to dynamic panel. CCE procedure is applicable to panels with single or multiple unobserved factors so long as the numbers of unobserved factors is fixed. The above mentioned estimations are dealing with the static case, for dynamic case new studies appear in the literature. Sarafidis and Robertson (2009) prove that also standard dynamic panel IV and GMM estimators either in levels or first difference are consistent as $N \rightarrow \infty$ for fixed T as the moment conditions applied by these estimators are invalid under cross sectional dependence. Groote and Everaert (2010) found that POLS estimator is also inconsistent for $T \rightarrow \infty$ as the temporal dependence in the unobserved factor implies correlation with lagged dependent variable. Groote and Everaert (2010) show that for dynamic case PCCE consistent if and only if $N, T \rightarrow \infty$. On the other hand, they have proposed an other estimator which is called restricted PCCE. The restricted version preferable due to its unrestricted version for N large T fixed version. Groote and Everaert (2010) suggest that the PCCE estimator is quite useful for estimating cross-sectional dependent dynamic panel data models provided T is not too small. Furthermore, Pesaran (2006) showed by Monte Carlo experiment that the simulation results favor the CCEP estimator for small to moderate sample sizes and slightly favor CCEMG when N and T are relatively large. This study mentioned CCEP estimator in small samples may be related to this issue⁷. Hence, this estimation process does not deal with the endogeneity bias in NDPCCE estimation⁸.

⁶In Pesaran and Smith (1995) heterogeneity comes from the random coefficient models. Nonlinear estimation procedure Gonzales *et al.* (2005) may capture these random coefficients in two different regimes.

⁷Smith and Fuertes (2010) classified the heterogeneity bias in large T sample structure.

⁸For DPCCE estimation Sarafidis and Yamagata (2010) investigate the properties of instrumental variable estimation and compare their estimators with GMM. However, they deal with the linear model where this estimator is more efficient and constitute consistent estimators.

Biases			
Nickell	Heterogeneity	CSD*	Endogeneity
Remedies			
IV, GMM	MG, PMG, RCM**	CCE, SURE- GLS***	IV, GMM
For N small T moderate or large			
Not prevail: X	Prevail: $\sqrt{\quad}$	Prevail: $\sqrt{\quad}$	Prevail: $\sqrt{\quad}$
Nonlinear Panel estimation			
X	X	$\sqrt{\quad}$	$\sqrt{\quad}$ ****
Nonlinear CCE Panel estimation			
X	X	X	X *****

*CSD denotes Cross Section Dependency

** We can include Swamy model as well. MG denotes mean group, PMG pooled mean group, RCM denotes random coefficient model.

*** There are other methods such as time effects/ demeaning, orthoganilization, PANIC Panel Analysis of Non-stationarity in the Idionsyncratic and Common components and residual principal component methods

**** $\sqrt{\quad}$ means nonlinear model partially correct endogeneity bias (Fouquau et al. 2008).

***** CCE estimation further corrected this bias Omay and Kan (2010).

3. Concluding Remarks

This survey intends to review the nonlinear panel data analysis with respect to biases and model structures. The flourishing literature of nonlinear panel data models introduces new remedies and difficulties to the panel data analysis. Fortunately, the solutions are found hastily as documented in this survey. For the future study this review can be extended by the new developments in this area.

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Cross-section Dependency and the Effects of Nonlinearity in Panel Unit Testing

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Abstract

In this study, we have analyzed the Cross Section Dependence (CSD) problem that is frequently encountered in a panel unit root setting by using the Pesaran (2004, 2008) CD tests. For this purpose we have generated cross sectionally dependent data and investigated the effects of nonlinear modeling on the cross section dependency problem inherited in panel analysis. The simulation study shows us that the nonlinear models remedy some part of this CS.

Keywords: Panel Unit Root; Cross Section Dependency Bias; Cross Section Dependency Test.

Jel Classification: C1,C12

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1. Introduction

In this study we are investigating the effects of nonlinear model structure for remedying the cross section dependence (CSD) problem. Cross section dependency is mainly a problem of large time dimension in the panel data analysis and may arise due to spatial correlations, spill-over effects, omitted global/multilateral variables, common unobserved shocks or general residual interdependence. In the existence of cross-sectionally correlated error terms, traditional OLS-based estimations are inefficient and nullify much inferential theory of panel data models. In presence of CSD, SURE-GLS (Seemingly Unrelated Regression Equations and Generalized Least Squares) can be used as a remedy and is feasible when the cross-section dimension N is smaller than the time series dimension T . Omay *et al.* (2014) used the SURE-GLS approach for the CSD bias. They have demonstrated that the system SURE-GLS eliminates the cross sectional dependence and endogeneity bias together. The typical approach is to consider the equations from the different cross-sectional units as a system of seemingly unrelated regression equations and then estimate the system by Generalized Least Squares (GLS) method (Omay *et al.*, 2014). If both of these dimensions (N and T) are similar, the disturbance covariance matrix will be rank deficient (Omay and Kan, 2010). Nevertheless, when the covariances between the errors of different cross-sectional units are non-zero due to common omitted variables, it is not obvious that SURE-GLS is always the right approach Coakley *et al.*(2002). The other popular approach is Pesaran's (2006) common correlated effect estimator approach, which suggests a method that makes use of cross-sectional averages to offer valid inference for stationary panel regressions with multifactor error structure. On the other hand, Pesaran (2007) proposes a new panel unit root test (called as the CADF statistic) that tackles the cross sectional dependence problem by using CC estimator and shows that the standard IPS test that does not allow for cross-sectional dependence can be seriously biased if the degree of cross-section dependence is sufficiently large². We know that, remedying cross sectional dependence across individual units is crucial in developing a panel unit root test because ignoring cross sectional dependence would create important power losses and size distortions (O'Connell, 1998)

There are some recent attempts to show the bias reduction properties of different remedies. One of them is the study by Kapetanios *et al.* (2011). They show that the CCE estimators have better small sample properties than the factor-based estimators. Besides, Westerlund and Urbain (2011) have also presented that the CCE estimator is less biased than the principal component estimator, although these two estimators have the equal variances. Furthermore, Pesaran and Tosetti (2011) have proven that the CCE estimator yields consistent estimates of the slope coefficients for the panel data model with both the multifactor error structure and the spatial error correlation. All these studies have used an estimation of parameter in interest strategy which involves explicitly estimating the coefficient and deciding whether it is biased or not. Fortunately, the other method may be the usage of CD test for showing the bias reduction in the panel unit root test. The first cross-section dependence test was proposed by Pesaran (2004). By using this test and the Pesarans' adjusted CD-LM test; we aim to show the reduction of CSD bias in panel unit root tests when nonlinear structures are introduced into the testing process. Omay and Kan (2010) have shown this bias reduction in the case of static nonlinear panel data model, namely the heterogeneous panel smooth transition model (PSTR). Specifically, they have documented that in each stage (linear model,

² Phillips and Sul (2003), Bai and Ng (2004), Moon and Perron (2004), Bai and Carrion-i-Silvestre (2009) have applied the principal component analysis to deal with cross sectional correlations. Alternatively, the bootstrap-based panel unit root tests have been suggested to deal with the general structure of cross sectional correlations, e.g., Maddala and Wu (1999), Chang (2004), Smith *et al.* (2004) and Ucar and Omay (2009).

linearized model, nonlinear model and the nonlinear model plus CCE estimation), the cross-sectional dependence diminishes. According to these authors the progress of the test statistics reveals that non-linear estimation eliminates some part of the cross-section dependence due to model misspecification. Furthermore, when they apply the CCE estimator, the cross-sectional dependence is fully removed from their PSTR estimations. Depending on the findings of Omay and Kan (2010) we extend this study to the panel unit root testing. From our simulation study we have also shown that that non-linear estimation eliminates some part of the cross-sectional dependence due to model misspecification. The model misspecification is CSD if we estimate the panel unit root test by using the cross sectional independent panel assumption such as IPS. However, when an Ucar and Omay (2009) type of nonlinearity is taken into account, some part of the CSD that is remedied this time imposes pseudo nonlinearity.

The rest of the paper is organized as follows. Section 2 gives the CD-LM tests along with the Monte-Carlo experiment design and the simulation results. Section 3 concludes.

2. The simulation study and the CD tests

In this section, we introduce the CD-LM test of the CSD bias. Pesaran (2004) shows that his cross-sectional dependence test (CD test) can also be applied to models including small or large N and T . Moreover, this diagnostic CD test does not require an a priori specification of connection or spatial matrix. The CD test is based on a simple average of all pair-wise correlation coefficients of the OLS residuals from the individual regressions in the panel:

$$\Delta y_{it} = \mu_i + \beta_i' x_{it} + u_{it} \quad (1)$$

where, on the time domain $t = 1, 2, \dots, T$, for the cross-section units $i = 1, 2, \dots, N$. x_{it} is a $k \times 1$ vector of observed time-varying regressors. The individual intercepts, μ_i and slope coefficients β_i are defined on a compact set permitted to vary across i . For each i , $u_{it} \sim iid(0, \sigma_{i,u}^2)$ and for all t , although they could be cross-sectionally correlated. The sample estimate of the pair-wise correlation in the residuals is:

$$\hat{\rho}_{ij} = \hat{\rho}_{ji} = \frac{\sum_{t=1}^T e_{it} e_{jt}}{\left(\sum_{t=1}^T e_{it}^2 \right)^{1/2} \left(\sum_{t=1}^T e_{jt}^2 \right)^{1/2}} \quad (2)$$

while the e_{it} is the OLS estimates of u_{it} , defined as:

$$e_{it} = \Delta y_{it} - \hat{\mu}_i - \hat{\beta}_i' x_{it} \quad (3)$$

The proposed CD test by Pesaran (2004) yields:

$$CD = \sqrt{\frac{2T}{N(N-1)}} \left(\sum_{i=1}^{N-1} \sum_{j=i+1}^N \hat{\rho}_{ij} \right) \quad (4)$$

The second CD test, which is used in Pesaran (2004), is proposed by Breusch and Pagan (1980):

$$CD_{LM1} = T \left(\sum_{i=1}^{N-1} \sum_{j=i+1}^N \hat{\rho}_{ij} \right) \quad (5)$$

following the scaled version of CD_{LM1} which can be considered for testing the hypothesis of cross-sectional dependence even for large N and T:

$$CD_{LM2} = \sqrt{\frac{1}{N(N-1)}} \left(\sum_{i=1}^{N-1} \sum_{j=i+1}^N (T \cdot \hat{\rho}_{ij}^2 - 1) \right) \quad (6)$$

By using these CD tests we organize some Monte Carlo experiments. The first design is as follows:

Linear model with single factor:

$$y_{it} = (1 - \phi_i) \mu_i + \phi_i y_{i,t-1} + \eta_{it} \quad (7)$$

where

$$\eta_{it} = \gamma_i f_t + u_{it} \quad (8)$$

and $u_{it} \sim N(0, \sigma_i^2)$ with $\sigma_i^2 \sim iidU[0.5, 1.5]$

Notice here that f_t is a factor variable that affects dependent and independent variables, respectively. We used this data generating process and apply the IPS test, UO test and UO test with CCE estimation and obtain CD-LM tests. We have used the UO test because Omay and Kan (2010) have used the linearized version of the PSTAR and obtained a bias reduction in the CD-LM test. This indicates that any nonlinear structure included into the panel unit root test under cross-sectionally dependent panels reduce some part of the CSD imposed by factor variable.

Table 1. CD test for different remedies

T/N	5			25			100		
	NR	LRR	LRRCC	NR	LRR	LRRCC	NR	LRR	LRRCC
30	2.027	1.999	-3.891	22.029	21.724	-3.412	64.888	64.107	-3.291
50	2.728	2.688	-5.219	29.046	28.711	-4.612	85.281	84.397	-4.471
70	3.260	3.212	-6.271	34.666	34.274	-5.561	101.743	100.727	-5.403
100	3.865	3.809	-7.574	41.718	41.279	-6.737	122.173	121.026	-6.561
150	4.785	4.722	-9.347	51.225	50.744	-8.336	149.943	148.674	-8.124
200	5.549	5.477	-10.845	59.383	58.832	-9.678	173.605	172.282	-9.428

Note: NR means not remedied, we use only IPS test while obtaining the residuals, LRR means the linearized version of the nonlinear model. We use the UO test for linearized version. LRRCC means the linearized version with CCE estimator.

As it can be seen readily from Table 1, the CD test is decreasing in every step, which means that the nonlinearity imposed into the model limits the potential cross section dependency bias. This finding lends a support for the results obtained by Omay and Kan (2010) for the nonlinear static panel data model.

3. Conclusion

In this study we have investigated the cross-sectionally dependent panels and the effects of nonlinear modeling on cross-sectional dependence problem inherited in panel analysis. The simulation study shows us that the nonlinear models remedy some part of the CSD in dynamic panel data models. For future research these findings can be analyzed by using different data generating processes and remedies.

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